

Unfolding of event-by-event net-charge distributions in heavy-ion collision

P. Garg¹, D. K. Mishra², P. K. Netrakanti², A. K. Mohanty²
and B. Mohanty³

¹ Department of Physics, Banaras Hindu University, Varanasi 221005, India

² Nuclear Physics Division, Bhabha Atomic Research Center, Mumbai 400094, India

³ School of Physical Sciences, National Institute of Science Education and Research, Bhubaneswar 751005, India

Abstract.

We discuss a method to obtain the true event-by-event net-charge multiplicity distributions from a corresponding measured distribution which is subjected to detector effects such as finite particle counting efficiency. The approach is based on the Bayes method for unfolding of distributions. We are able to faithfully unfold back the measured distributions to match with their corresponding true distributions obtained for a widely varying underlying particle production mechanism, beam energy and collision centrality. Particularly the mean, variance, skewness, kurtosis, their products and ratios of net-charge distributions from the event generators are shown to be successfully unfolded from the measured distributions constructed to mimic a real experimental distribution. We demonstrate the necessity to account for detector effects before associating the higher moments of net-charge distributions with physical quantities or phenomena. The advantage of this approach being that one need not construct new observable to cancel out detector effects which loose their ability to be connected to physical quantities calculable in standard theories.

PACS numbers: 25.75.Gz, 12.38.Mh, 21.65.Qr, 25.75.-q, 25.75.Nq

1. Introduction

Higher moments of the event-by-event distribution of conserved quantities like net-charge, net-baryon number and net-strangeness in heavy-ion collisions have been found to be useful observable to characterize the system formed in the collisions [1]. Higher moments have been shown to be related to the correlation length [2] and susceptibilities [3, 4] of the system and hence can be used to look for signals of phase transition and critical point [5, 6, 7]. They have also been shown to be useful for studying the bulk QCD thermodynamics at high temperature [8]. Specifically, proposals have been made to extract the freeze-out properties of the system using higher moments of net-charge and net-baryon number distributions, in a way very similar to that done using the particle yields and ratios [9, 10, 11].

Any experimental measurement is susceptible to the effects such as the finite acceptance, finite efficiency of counting the number of particles produced in the collisions and other background effects [12]. It is almost impossible to know some of these quantities for each event so as to correct for the effects in an event-by-event distribution. Hence most of the experimentally measured event-by-event distributions are presented without these corrections [1, 13, 14, 15]. These corrections are carried out on an average level for reporting the yields of the produced particles (typically the first moment of the multiplicity distributions) [12]. Comparison of uncorrected experimental event-by-event distributions to theoretical calculations needs to be done carefully. For example, using the corrected mean multiplicities to explain the uncorrected measured event-by-event distributions could lead to different conclusions [16, 17].

Judicious construction of event-by-event observables have been proposed to cancel out detector effects to first order [18, 19, 20, 21]. However, while making these constructs, one may sometimes loose the ability to compare them to the theoretically calculated quantities in order to extract meaningful physical insights. That introduce additional complexities which makes it difficult for a proper physical interpretation of the observable. As an example, the moments of the multiplicity distribution of conserved quantities can be shown to be proportional to correlation length (ξ) of the system. The variances ($\sigma^2 \equiv \langle(\Delta N)^2\rangle$; $\Delta N = N - M$; M is the mean) of these distributions are related to ξ as $\sigma^2 \sim \xi^2$, skewness ($S = \langle(\Delta N)^3\rangle/\sigma^3$) goes as $\xi^{4.5}$ and kurtosis ($\kappa = [\langle(\Delta N)^4\rangle/\sigma^4] - 3$) goes as ξ^7 [2]. Their product such as $\kappa\sigma^2$ are related to the ratio of fourth order ($\chi^{(4)}$) to second order ($\chi^{(2)}$) susceptibilities [3, 4]. Where $\chi^{(2)} = \frac{\langle(\Delta N)^2\rangle}{VT}$, V is the volume, and ΔN could be the net-baryon number or net-charge number. In order to cancel out the acceptance and efficiency effects to first order for these observables, constructs such as normalized factorial moments (defined later) can be made. The factorial moments of a particular order however become complicated function of lower order moments. Thereby making their interpretation difficult in terms of physical observables such as ξ or χ calculated in a standard theory.

Here we give a simple calculation to illustrate our point of view. Let N represents the produced multiplicity and n being the actually measured multiplicity in an

experiment. We parametrize the detector response in the experiment by a binomial probability distribution function given by,

$$B(n : N, \epsilon) = \frac{N!}{n!(N-n)!} \epsilon^n (1-\epsilon)^{N-n}, \quad (1)$$

where ϵ is the particle counting efficiency.

We further consider that the produced multiplicity follows probability distribution function $P(N)$, and that for measured distribution is $P(n)$. Then the mean of measured multiplicity distribution $\langle n \rangle$ can be related to the mean of the actually produced multiplicity distribution as,

$$\begin{aligned} \langle n \rangle &= \int n P(n) dn = \int n dn \int B(n | N) P(N) dN \\ &= \int P(N) dN \int B(n | N) n dn = \epsilon \int P(N) N dN = \epsilon \langle N \rangle. \end{aligned} \quad (2)$$

Similarly it can be shown that,

$$\langle n^2 \rangle = \epsilon(1-\epsilon) \langle N \rangle + \epsilon^2 \langle N^2 \rangle. \quad (3)$$

Now let us suppose that we can correct event-by-event particle counting efficiency, the variance of the resultant measured distribution can be shown to be,

$$\sigma^2(n/\epsilon) = \frac{1-\epsilon}{\epsilon} \langle N \rangle + \sigma^2(N) \quad (4)$$

We find that the variance of n/ϵ is not equal to the variance of N even though the mean of n/ϵ is equal to the mean of N . Similar derivations and conclusions can be done for higher order moments.

Alternatively, one can construct second order factorial moments such as

$$\frac{\langle n(n-1) \rangle}{\langle n \rangle^2} = \frac{\epsilon^2 \langle N(N-1) \rangle}{\epsilon^2 \langle N \rangle^2} = \frac{\langle N(N-1) \rangle}{\langle N \rangle^2}, \quad (5)$$

and the fourth order factorial moment as,

$$\frac{\langle n(n-1)(n-2)(n-3) \rangle}{\langle n \rangle^4} = \frac{\langle N(N-1)(N-2)(N-3) \rangle}{\langle N \rangle^4}. \quad (6)$$

These are found to be independent of efficiency effects. In these we also assume that ϵ does not vary event-by-event.

However a closer look at these construct will reveal that,

$$\frac{\langle n(n-1) \rangle}{\langle n \rangle^2} = \frac{\sigma^2}{\langle n \rangle^2} - \frac{1}{\langle n \rangle} + 1 \quad (7)$$

and

$$\begin{aligned} \frac{\langle n(n-1)(n-2)(n-3) \rangle}{\langle n \rangle^4} &= 11 \frac{\sigma^2}{\langle n \rangle^4} + \frac{1}{\langle n \rangle^2} \\ &- 6S \frac{\sigma^3}{\langle n \rangle^4} - 18 \frac{\sigma^2}{\langle n \rangle^3} + \kappa \frac{\sigma^4}{\langle n \rangle^4} \\ &+ 4S \frac{\sigma^3}{\langle n \rangle^3} - 3 \frac{\sigma^2}{\langle n \rangle^2} - 2 - \frac{6}{\langle n \rangle^3}. \end{aligned} \quad (8)$$

While trying to remove the detector effects we have arrived at constructs which loose the purity of moments or become involved functions of lower order moments. Thereby making it difficult to directly connect to physical observables such as susceptibilities or their ratios which can give important insight to the bulk properties of the system formed in heavy-ion collisions.

Keeping in view the importance of higher moments of multiplicity distributions to characterize the system formed in heavy-ion collisions, it is necessary to have a proper way to compare the measurements and theory calculations. At the same time ensure that experimental artifacts like acceptance and particle counting efficiency are removed. In this paper, we propose an approach based on unfolding of the measured (actually measured in experiments) multiplicity distribution to get back the true (actually produced in the collisions) distributions produced in the collisions. Such a method seems to work only if the detector response can be satisfactorily modeled and the statistics is large enough.

The paper is organized as follows. In the next section we discuss the event generators used in this study. In section 3 we discuss method of unfolding. In section 4 we present the results for the moments of the net-charge distribution as a function of collision centrality (defined in terms of the number of participating nucleons, N_{part}). A brief discussion of the limitations of the approach is also presented. Finally we summarize our study in section 5.

2. Event generators

In this study we have used two event generators HIJING [22] (version 1.37) and THERMINATOR [23] (version 2.0). They provide the possibility of different probability distribution for charged particle multiplicity, to study our proposal. While HIJING distributions are based on the physics due to QCD inspired models for multiple jet production, the THERMINATOR distributions are based on systems in thermodynamical equilibrium. The further details of the models can be found in Ref. [22] for HIJING and in Ref. [23] for THERMINATOR. For HIJING the events were generated with default settings and jet quenching on, while for THERMINATOR the default settings were used. We only focus on the net-charge distributions within a realistic acceptance of the current experiments at RHIC, that is pseudorapidity range between $-0.5 < \eta < 0.5$, transverse momentum range between $0.2 < p_T < 2.0$ GeV/ c with full azimuthal coverage. The analysis is carried out for 19.6 GeV Au+Au collisions using the events from HIJING model and 200 GeV Au+Au collisions using the events from the THERMINATOR model as a function of collision centrality. About 5 million events are produced for each centrality studied in both the event generators. We have checked that the conclusions from each model at other energies are similar to that presented in this paper. Such a combination of model and beam energy is an arbitrary choice done to reflect a wide range of kinematics and physics of particle production. The average charge particle multiplicity counting efficiency is taken to be 65% following the efficiency

as a function of p_T available for charged pions in Ref. [12].

3. Bayes method for unfolding of distributions

The Bayes unfolding algorithm of RooUnfold package is used in general to remove the effects of measurement resolutions, systematic biases and detection efficiency to determine the true distributions [24]. The RooUnfoldBayes algorithm based on Bayes theorem uses the method described by D'Agostini in Ref. [25].

The procedure of Bayes unfolding can be explained by the causes C and effects E . In our study, *causes* correspond to the true multiplicity values and *effects* to the measured multiplicity values which are affected by the inefficiencies. If one observes $n(E)$ events with effect E due to several independent causes ($C_i, i = 1, 2, \dots, n_C$) then the expected number of events assignable to each of the causes is given by:

$$\hat{n}(C_i) = n(E)P(C_i|E) \quad (9)$$

where

$$P(C_i|E) = \frac{P(E|C_i)P(C_i)}{\sum_{l=1}^{n_c} P(E|C_l)P(C_l)} \quad (10)$$

Now if we observe that the outcome of a measurement has several possible effects $E_j (j = 1, 2, 3, \dots, n_E)$ for a given cause C_i then the expected number of events to be assigned to each of the causes and only due to the observed events can be calculated to each effect by:

$$\hat{n}(C_i) = \sum_{j=1}^{n_E} n(E_j)P(C_i|E_j). \quad (11)$$

$P(C_i|E_j)$ is the probability that different causes C_i were responsible for the observed effect E_j and is calculated by Bayes theorem as:

$$P(C_i|E_j) = \frac{P(E_j|C_i)P_0(C_i)}{\sum_{l=1}^{n_c} P(E_j|C_l)P_0(C_l)} \quad (12)$$

where $P_0(C_i)$ are the initial probabilities. If we take into account the inefficiency then the best estimate of the true number of events is given by,

$$\hat{n}(C_i) = \frac{1}{\epsilon_i} \sum_{j=1}^{n_E} n(E_j)P(C_i|E_j) \quad \epsilon_i \neq 0 \quad (13)$$

where ϵ_i is the efficiency of detecting the cause C_i in any of the possible effects. If $\epsilon_i = 0$ then $\hat{n}(C_i)$ is set to zero, since the experiment is not sensitive to the cause C_i .

The above equation can be written in terms of unfolding or response matrix M_{ij} as,

$$\hat{n}(C_i) = \sum_{j=1}^{n_E} M_{ij}n(E_j) \quad (14)$$

The response matrix is constructed by repeated application of Bayes theorem and the regularization is achieved by stopping the iterations before reaching the "true" inverse. Further details of the procedure can be found in [25].

For the present study, 5M Au+Au collision events are produced for each centrality bin at $\sqrt{s_{NN}} = 19.6$ GeV and 200 GeV using HIJING and THERMINATOR event generators respectively. With these events, the *true* distribution of net-charge ($\Delta N = N^+ - N^-$) is constructed on an event-by-event basis. The positive (N^+) and negative (N^-) charge particles are selected for each event with transverse momentum range between $0.2 < p_T < 2.0$ GeV/c and pseudorapidity range between $-0.5 < \eta < 0.5$ with full azimuthal coverage.

The individual *true* N^+ and N^- are smeared with a Gaussian function with the mean value corresponding to the average efficiency of 65% as obtained from the parametrization of the p_T dependent efficiency for charged pions from STAR experiment [12]. The width of the Gaussian distribution is taken as 10% of the mean. The smeared N^+ and N^- distributions will be called as *measured* distributions. The measured net-charge distribution is then constructed with these *measured* N^+ and N^- distributions.

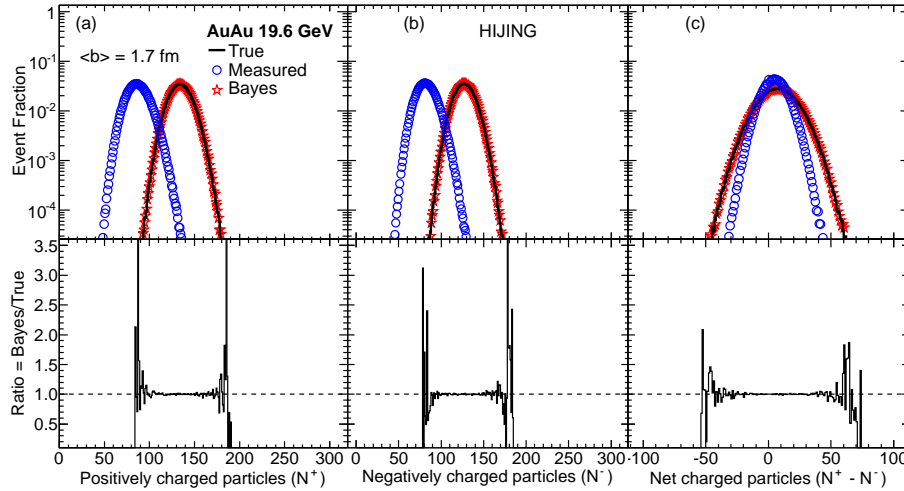


Figure 1. (Color online) Top panel: Event-by-event distribution of positive, negative and net-charge (denoted as “True”, solid line) in Au+Au collisions for impact parameter $b = 1.7$ fm at $\sqrt{s_{NN}} = 19.6$ GeV from HIJING event generator. Also shown are the corresponding distributions after applying acceptance and efficiency effects as discussed in the text (denoted as “Measured”, open circles). The unfolded distributions are shown as red stars and denoted as “Bayes”. Bottom panel: Shows the ratio of the unfolded to the True distributions.

To construct the response matrix for each centrality, 2.5M events are used as *training true* distribution of net-charge and rest of the events are used as *training measured* (after smearing on an event-by-event basis) distribution. The events for *training true* and *training measured* are selected separately to construct the response matrix, in order to avoid the effect of auto-correlation. It also uses the information

of an event that is not measured out of true distributions and is counted towards the inefficiency while constructing the response matrix.

The measured distribution of net-charge from the remaining 2.5M events is unfolded with response matrix obtained from the training procedure using iterative Bayes theorem. The number of iterations is called the regularization parameter. The present study uses the optimal value of 4 for the regularization. True, measured and unfolding are done for finer bins of each centrality and then combined to make 5% bin to eliminate the finite centrality bin-width effect. The moments of net-charge distributions are derived using cumulant method as described in [26] and are compared for true, measured and unfolded distributions.

4. Results and Discussions

Figure 1 shows the true, measured and unfolded distributions for positive charge (panel a), negative charge (panel b) and net-charge (panel c) for most central events corresponding to an average impact parameter of 1.7 fm of Au+Au collisions from HIJING at $\sqrt{s_{NN}} = 19.6$ GeV on an event-by-event basis. The true distributions are shown as solid lines, measured distributions (subjected to particle counting efficiency) are shown as blue open circles and the unfolded distributions denoted as “Bayes” are shown as red star. For all the cases, the respective true distributions are reproduced from the measured distribution using the unfolding technique. The bottom panel of Fig.1 shows the ratio of unfolded to the true distributions corresponding to the same distributions as shown in the respective top panels of Fig.1.

The ratio is close to unity within the statistical errors, suggesting that the unfolding procedure is able to get back the true distribution from a measured distribution which is subjected to inefficiencies in particle counting. Similar conclusions are obtained for distributions for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV from THERMINATOR, hence are not shown in this paper. From now onwards we will only concentrate on the net-charge distributions.

The four moments M , σ , S and κ of the net-charge distributions in Au+Au collisions at $\sqrt{s_{NN}} = 19.6$ GeV from the constructed true, measured and unfolded distributions as a function of centrality (N_{part}) are shown in Fig.2. The mean and standard deviation increases with N_{part} , while the skewness and kurtosis decreases with N_{part} . This is in accordance with the central limit theorem[1]. The mean and variance of the measured distribution are smaller compared to those of the true, as we have particle counting inefficiencies for the measured case. The unfolded moments are found to closely follow the corresponding values of their respective true distributions. This can be more clearly seen from the ratio plots in Fig.3. The value of the ratio of unfolded to true distribution as a function of N_{part} is around unity for all the four moments studied. Thus the unfolding method followed in this paper reproduces all the moments of true distribution from the measured distribution. Although not shown here, similar conclusions are obtained separately for the positive and negative charge particle

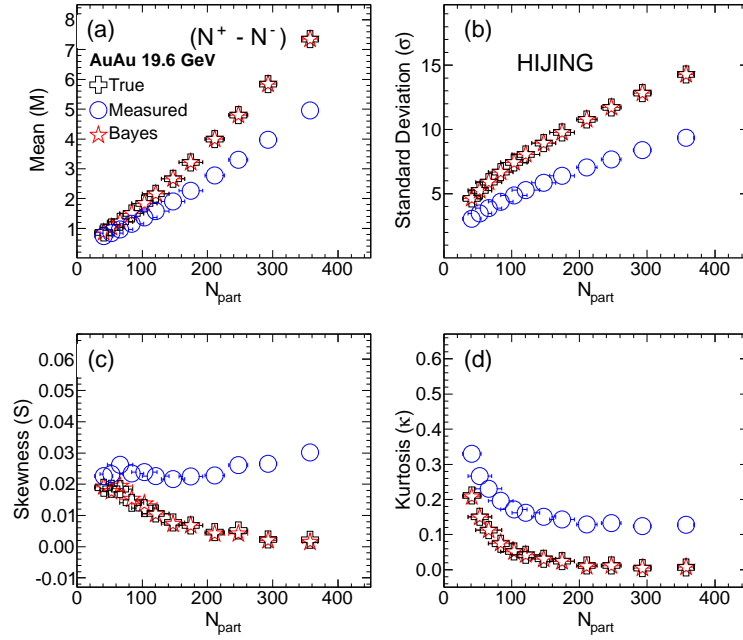


Figure 2. (Color online) Mean, standard deviation, skewness and kurtosis of net-charge distribution in Au+Au collisions at $\sqrt{s_{NN}} = 19.6$ GeV from HIJING event generator. Results are shown for the True, measured and Bayes unfolded distributions as a function of N_{part} .

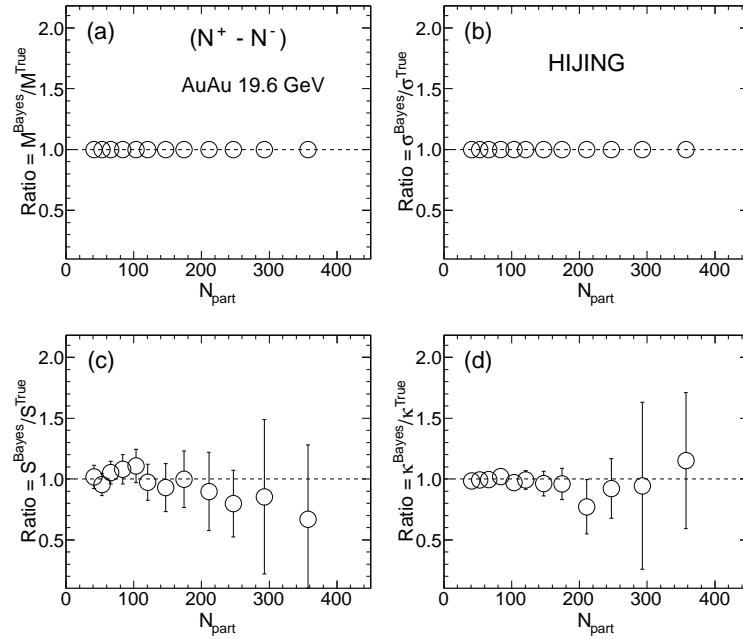


Figure 3. Ratio of the unfolded to the True net-charge distribution moments of Fig.2 as a function of N_{part} .

multiplicity distributions.

The centrality dependence of ratio of moments (σ^2/M) and product of moments ($S\sigma$ and $\kappa\sigma^2$) are shown in Fig.4. The importance of unfolding is clearly demonstrated

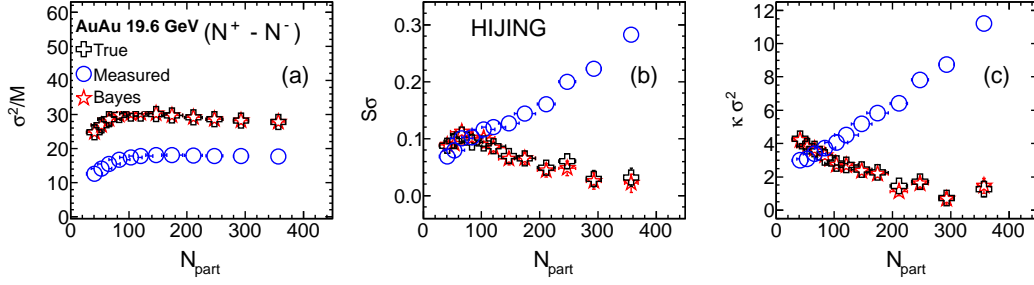


Figure 4. (Color online) Ratio (panel a) and product of moments (panel (b) and (c)) of net-charge distributions in Au+Au collisions at $\sqrt{s_{NN}} = 19.6$ GeV from HIJING event generator. The results are for the True, measured and Bayes unfolded distributions as a function of N_{part} .

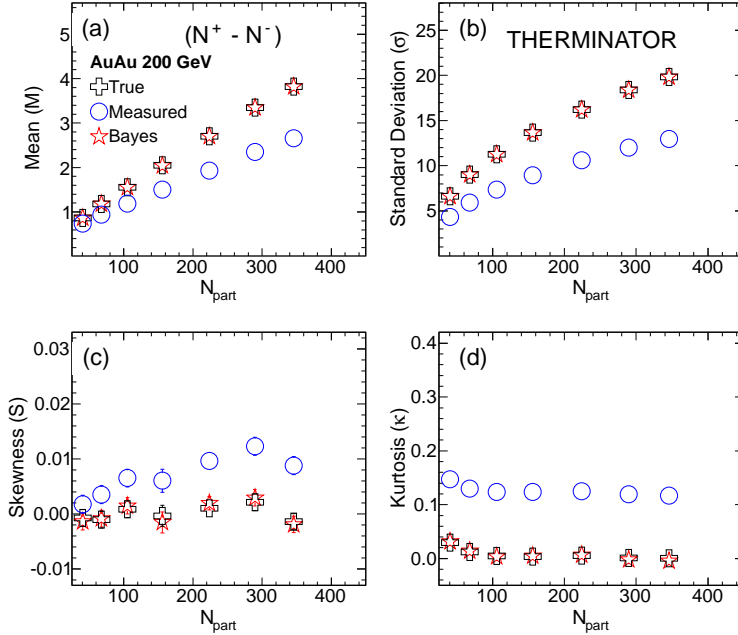


Figure 5. (Color online) Mean, standard deviation, skewness and kurtosis of net-charge distribution in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV from THERMINATOR event generator. Results are shown for the True, measured and Bayes unfolded distributions as a function of N_{part} .

by looking at the dependences of the ratio and product of moments on the N_{part} . While for the true distribution the product of moments decreases with N_{part} , those for measured actually has an opposite trend. Indicating any physics conclusions associated

with variation of $S\sigma$ and $\kappa\sigma^2$ with N_{part} for net-charge distributions could be highly misleading. However, very nice agreement between true and unfolded distributions are observed. They are nicely consistent even for the product of higher moments ($S\sigma$ and $\kappa\sigma^2$) which are very sensitive to the shape of the distributions. Suggesting that the unfolded distributions are well reproduced as the true distributions by using Bayes unfolding algorithm.

In order to validate the applicability of unfolding algorithm for different physics processes, a thermal model based THERMINATOR event generator is also used. Figure 5 shows the centrality dependence of various moments of net-charge distribution in Au+Au collisions at $\sqrt{s_{\text{NN}}} = 200$ GeV from the true, measured and unfolded distributions. The trends of the moments as a function of N_{part} is similar to that seen for HIJING (Fig. 2), although the magnitude of the moments are different. All the four moments of the unfolded distributions are well reproduce as the true distributions.

Figure 6 shows the σ^2/M , $S\sigma$ and $\kappa\sigma^2$ as a function of N_{part} of net-charge distributions from the true, measured and unfolded distributions. Here also, as was seen for the HIJING results (Fig. 4), the ratio and products of moments from unfolded distributions are reproduced as true distributions up to a good extent. This suggests that the method proposed in this paper works equally well for parent distributions produced from very different particle production mechanisms as well as over a wide range of beam energies.

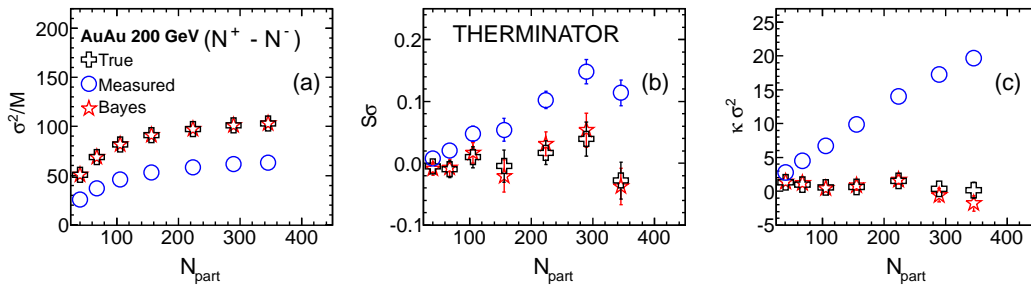


Figure 6. (Color online) Product of moments of net-charge distributions in Au+Au collisions at $\sqrt{s_{\text{NN}}} = 200$ GeV from THERMINATOR event generator. The results are for the True, measured and Bayes unfolded distributions as a function of N_{part} .

To study the effect of variation of efficiency on ratio and product of moments, the true distributions are smeared with a constant efficiency of 65% to obtain the measured distributions. Figure 7 and Fig 8 show the σ^2/M , $S\sigma$ and $\kappa\sigma^2$ as a function of N_{part} of net-charge distributions from the true, measured and unfolded distributions with constant efficiency in Au+Au collisions at $\sqrt{s_{\text{NN}}} = 19.6$ GeV and 200 GeV from HIJING and THERMINATOR event generators, respectively. Panel (a) of Fig 7 and Fig 8 shows similar effect as for event-by-event variation of efficiency (panel (a) of Fig 4 and Fig 6) on the σ^2/M of the measured distributions. The effect of constant efficiency on $S\sigma$ and $\kappa\sigma^2$ (panel (b) and (c) of Fig 7 and Fig 8) of measured distributions is small as

compared to event-by-event varying efficiency.

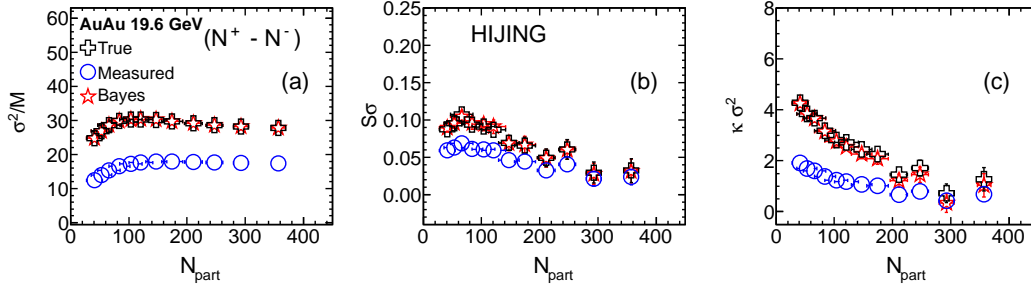


Figure 7. (Color online) Product of moments of net-charge distributions in Au+Au collisions at $\sqrt{s_{\text{NN}}} = 19.6$ GeV from HIJING event generator with constant efficiency of 65%. The results are for the True, measured and Bayes unfolded distributions as a function of N_{part} .

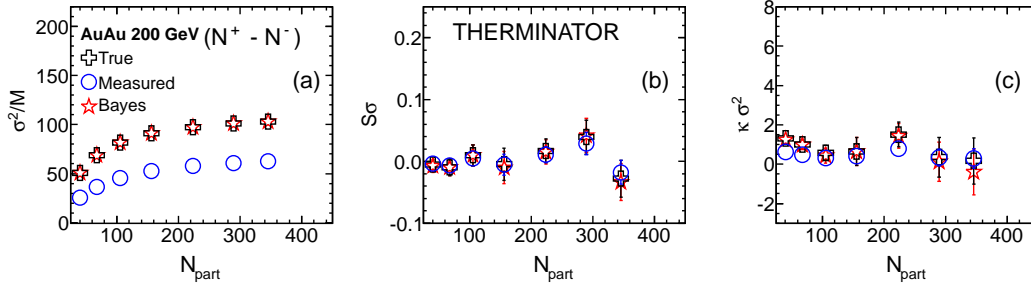


Figure 8. (Color online) Product of moments of net-charge distributions in Au+Au collisions at $\sqrt{s_{\text{NN}}} = 200$ GeV from THERMINATOR event generator with constant efficiency of 65%. The results are for the True, measured and Bayes unfolded distributions as a function of N_{part} .

In order to see the effect of energy dependence on our results we have carried out this study for net-charge distributions at midrapidity in 0-5% central Au+Au collisions in HIJING model for $\sqrt{s_{\text{NN}}} = 19.6, 27, 39, 62.4, 130$ and 200 GeV. The efficiency varies event-by-event as per a Gaussian distribution with mean of 65% and width of 10% of the mean. Figure 9 shows the Mean, standard deviation, skewness and kurtosis for the above system as a function of beam energy. The mean and variance of the measured distribution are smaller compared to those of the true, as we have seen for the centrality dependence study (Fig. 2 and Fig. 5). This is due the particle counting inefficiencies for the measured case. The unfolded moments are found to closely follow the corresponding values of their respective true distributions. Figure 10 shows the σ^2/M , $S\sigma$ and $\kappa\sigma^2$ as a function of $\sqrt{s_{\text{NN}}}$ of 0-5% Au+Au collisions net-charge distributions from the true, measured and unfolded distributions. Here also, as was seen for the centrality dependence results, the ratio and products of moments from unfolded distributions are

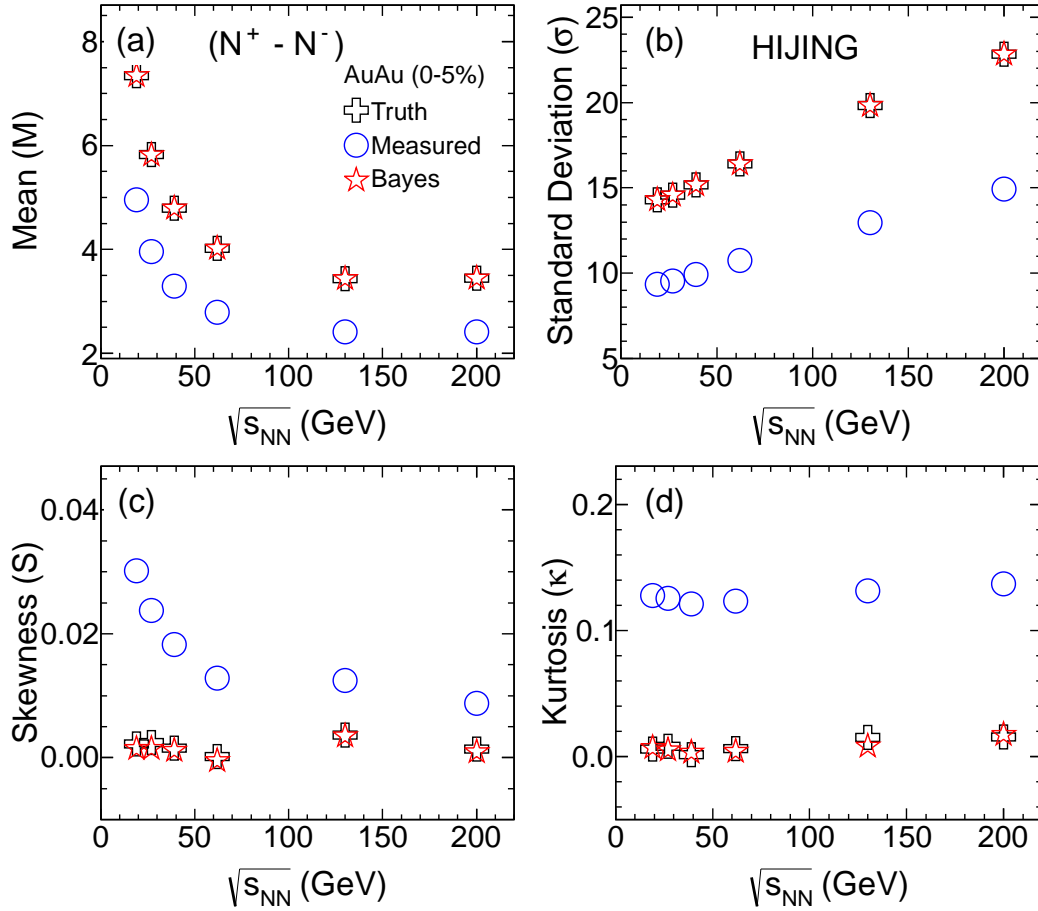


Figure 9. (Color online) Mean, standard deviation, skewness and kurtosis of net-charge distribution in 0-5% Au+Au collisions as a function of $\sqrt{s_{NN}}$ from HIJING event generator. Results are shown for the True, measured and Bayes unfolded distributions

reproduced as true distributions up to a good extent. This suggests that the method proposed in this paper works for parent distributions produced over a wide range of beam energies.

Our study shows that it is important to correct for event-by-event detector related effects before proper conclusions can be obtained from higher moments studies in heavy-ion collisions. We have provided a method of obtaining the true distributions through an unfolding technique. Such a method keeps the observables same and hence has the advantage of being used to compare to standard theory calculations. Although this procedure can be easily adapted to experimentally measured distributions, it has two important drawbacks. Unlike the current case, where we have used an event generator for the study and the true distribution is available for comparison, in a real experiment the true distribution is unknown. Hence it is very crucial that a realistic modeling of the detector response and particle production is available to obtain the response matrix for the unfolding calculations. In most cases, the modeling of the particle production and the detector response is highly event generator dependent and on how realistically the

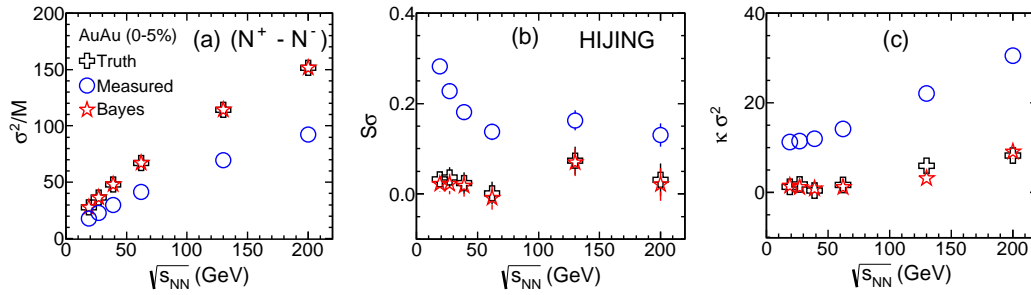


Figure 10. (Color online) Product of moments of net-charge distribution in 0-5% Au+Au collisions as a function of $\sqrt{s_{NN}}$ from HIJING event generator. The results are for the True, measured and Bayes unfolded distributions.

experimental conditions are simulated. The other disadvantage is, that the procedure works well for high event statistics as well as high average particle multiplicity per event. We have seen that large uncertainties enter into the unfolded distributions if we carry out this study with net-protons.

5. Summary

In summary, we have discussed a method to obtain the event-by-event true distributions of net-charge from the corresponding measured distributions which are subjected to detector effects like finite particle counting efficiencies. The approach used is based on the Bayes method for unfolding of distributions. We have used event generators HIJING and THERMINATOR to simulate the charged particle distributions produced in Au+Au collisions at $\sqrt{s_{NN}} = 19.6$ to 200 GeV respectively. The charge particle counting efficiency was varied by smearing the true distributions, on event-by-event basis using a Gaussian function with mean 0.65 and width 0.065, to construct the measured distributions. We have shown that the unfolded distribution has similar mean, variance, skewness and kurtosis as the true distributions for all the collision centralities studied. The product of the moments σ^2/M , $S\sigma$ and $\kappa\sigma^2$ which show an opposite trends versus N_{part} for the measured distributions compared to the true distributions are faithfully unfolded back to agree with the true distributions. For cases where the efficiency of charged particle counting is constant for all events, the differences between the measured and true are small for the product $S\sigma$ and $\kappa\sigma^2$ compared to the ratio σ^2/M . The unfolding process is demonstrated to work for distributions obtained from widely differing physical mechanism for production of charge particle and over a wide range of beam energy and collision centrality. They also work for the cases where the charged particle counting efficiencies vary event-by-event as well as for the case where the efficiencies are constant.

This method has some limitations, in terms of need for a proper modeling of the detector response and works well for high multiplicity and high event statistics dataset.

However the main advantage of this method is that we do not have to construct new observables which cancels out the detector effects. As the new constructs are usually subjected to difficulties in physical interpretation and cannot be directly compared to standard theoretical calculations.

Acknowledgments

Financial assistance from the Department of Atomic Energy, Government of India is gratefully acknowledged. BM is supported by the DST SwarnaJayanti project fellowship. PG acknowledges the financial support from CSIR, New Delhi, India.

- [1] M. M. Aggarwal *et al.* [STAR Collaboration], Phys. Rev. Lett. **105**, 022302 (2010) [arXiv:1004.4959 [nucl-ex]].
- [2] M. A. Stephanov, Phys. Rev. Lett. **102**, 032301 (2009) [arXiv:0809.3450 [hep-ph]].
- [3] A. Bazavov *et al.* [HotQCD Collaboration], Phys. Rev. D **86**, 034509 (2012) [arXiv:1203.0784 [hep-lat]].
- [4] M. Cheng, P. Hengde, C. Jung, F. Karsch, O. Kaczmarek, E. Laermann, R. D. Mawhinney and C. Miao *et al.*, Phys. Rev. D **79**, 074505 (2009) [arXiv:0811.1006 [hep-lat]].
- [5] M. A. Stephanov, Phys. Rev. Lett. **107**, 052301 (2011) [arXiv:1104.1627 [hep-ph]].
- [6] M. Asakawa, S. Ejiri and M. Kitazawa, Phys. Rev. Lett. **103**, 262301 (2009) [arXiv:0904.2089 [nucl-th]].
- [7] Y. Hatta and M. A. Stephanov, Phys. Rev. Lett. **91**, 102003 (2003) [Erratum-ibid. **91**, 129901 (2003)] [hep-ph/0302002].
- [8] S. Gupta, X. Luo, B. Mohanty, H. G. Ritter and N. Xu, Science **332**, 1525 (2011) [arXiv:1105.3934 [hep-ph]].
- [9] R. V. Gavai and S. Gupta, Phys. Lett. B **696**, 459 (2011) [arXiv:1001.3796 [hep-lat]].
- [10] F. Karsch and K. Redlich, Phys. Lett. B **695**, 136 (2011) [arXiv:1007.2581 [hep-ph]].
- [11] B. Friman, F. Karsch, K. Redlich and V. Skokov, Eur. Phys. J. C **71**, 1694 (2011) [arXiv:1103.3511 [hep-ph]].
- [12] B. I. Abelev *et al.* [STAR Collaboration], Phys. Rev. C **79**, 034909 (2009) [arXiv:0808.2041 [nucl-ex]].
- [13] H. Appelshauser *et al.* [NA49 Collaboration], Phys. Lett. B **459** (1999) 679 [hep-ex/9904014].
- [14] M. M. Aggarwal *et al.* [WA98 Collaboration], Phys. Rev. C **65** (2002) 054912 [nucl-ex/0108029].
- [15] K. Adcox *et al.* [PHENIX Collaboration], Phys. Rev. Lett. **89** (2002) 082301 [nucl-ex/0203014].
- [16] P. Braun-Munzinger, B. Friman, F. Karsch, K. Redlich and V. Skokov, Phys. Rev. C **84**, 064911 (2011) [arXiv:1107.4267 [hep-ph]].
- [17] B. Mohanty, Talk given at CPOD2011, Wuhan, China. Presentation at CPOD 2011
- [18] S. Mrowczynski, Phys. Lett. B **465** (1999) 8 [nucl-th/9905021].
- [19] S. A. Voloshin, V. Koch and H. G. Ritter, Phys. Rev. C **60**, 024901 (1999) [nucl-th/9903060].
- [20] A. Bialas, Phys. Rev. C **75**, 024904 (2007) [hep-ph/0701074].
- [21] C. Pruneau, S. Gavin and S. Voloshin, Phys. Rev. C **66**, 044904 (2002) [nucl-ex/0204011].
- [22] M. Gyulassy and X. -N. Wang, Comput. Phys. Commun. **83**, 307 (1994) [nucl-th/9502021].
- [23] A. Kisiel, T. Taluc, W. Broniowski and W. Florkowski, Comput. Phys. Commun. **174**, 669 (2006) [nucl-th/0504047].
- [24] T. Adye, [arXiv:1105.1160 [physics.data-an]]. RooUnfold package
- [25] G. D' Agostini, Nucl. Instrum. Meth. A **362**, 487 (1995).
- [26] X. Luo, J. Phys. G **39**, 025008 (2012) [arXiv:1109.0593 [nucl-ex]].